Software sotid for computation of site displacements due to the solid Earth tides

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Abstract:

Package sotid *provides a library for computing displacements due to the solid Earth tides using the frequency domain approach. It uses the HW95 expansion of the tide-generating potential.* sotid *allows us to compute solid Earth tide displacements with the precision up to 10 microns, what is much greater than the accuracy of Love numbers. Currently,* sotid *supports seven models of Love numbers of the 2-nd degree and one model of Love numbers of the 3-rd degree. When the new models of Love numbers become available they can be easily included in the package.* sotid *is written in Fortran90 and provides an interface for programs written in both Fortran and C.*

1 How to use

Four subroutines are to be called for computing site displacements due to sotid Earth tides:

1. sotid_set sets configuration of sotid. It specifies which model of Love numbers of the second and third degree should be used, which Love numbers should be assign to the zero frequency (permanent tide), whether all generalized Love numbers should be used, whether the tides of all orders of the second degrees should be taken into account.
2. sotid_pre computes intermediary quantities which do not depend on time, but depend on site positions. This subroutine is called only once.
3. sotid_tim computes intermediary quantities which depend on time, but do not depend on site coordinates. This subroutine is called for each moment of time under consideration.
4. sotid_dsp computes site displacements due to solid Earth tides for the moment of time defined in call sotid_tim, for the site defined in call sotid_pre using the models defined in call sotid_set.

Additional routine sotid_inq returns information about sotid configuration which was set in call sotid_set.

Interface to external sotid routines is defined in module SOTID_MODULE, constants are defined in sotid_data.i file.

2 Interface

2.1 SOTID_SET

```
! ************************************************************************
```

```
! *                                                                    *
! *   Routine  SOTID_SET  sets configuration parameters of SOTID       *
! *   software. Values of each configuration parameter are checked and *
! *   if the check is OK, SOTID_SET puts it in the field of TIDCNF.    *
! *                                                                    *
! * ________________________ Input parameters:  ________________________ *
! *                                                                    *
! *       GEN_LOVE ( INTEGER*4 ) -- Which generalized Love numbers are *
! *                                  to be used. Suppported values:    *
! *                               SOTID__PRN_ONLY                      *
! *                               SOTID__PL_ONLY                       *
! *                               SOTID__GEN_ALL                       *
! *       MODEL_2D ( INTEGER*4 ) -- Which model for the Love numbers of *
! *                                  the second degree is to be applied. *
! *                               SOTID__LOVE                          *
! *                               SOTID__IERS92                        *
! *                               SOTID__MDG97EL                       *
! *                               SOTID__MDG97AN                       *
! *                               SOTID__DDW99EH                       *
! *                               SOTID__DDW99IN                       *
! *                               SOTID__MAT00                         *
! *                               SOTID__MAT01                         *
! *       ORDER_2D ( INTEGER*4 ) -- Tidal waves of which order of the  *
! *                                  second degree should be taken into *
! *                                  account: zonal, diurnal,          *
! *                                  semi-dirunal or their combination. *
! *                                  NB: SOTID__2D_012ORD should be    *
! *                                  normally used for computation. Other *
! *                                  values are reserved for testing or *
! *                                  for a special purpose.            *
! *                               SOTID__2D_NONE                       *
! *                               SOTID__2D_0ORD                       *
! *                               SOTID__2D_01ORD                      *
! *                               SOTID__2D_02ORD                      *
! *                               SOTID__2D_012ORD                     *
! *                               SOTID__2D_1ORD                       *
! *                               SOTID__2D_12ORD                      *
! *                               SOTID__2D_2ORD                       *
! * ZERO_FREQ_LOVE ( INTEGER*4 ) -- Which Love numbers for the tides   *
! *                                  of the zero frequency should be used.*
! *                               SOTID__ZF_ZERO                       *
! *                               SOTID__ZF_LOVE                       *
! *                               SOTID__ZF_MDG97EL                    *
! *                               SOTID__ZF_MDG97AN                    *
! *                               SOTID__ZF_FLUI                       *
! *       MODEL_3D ( INTEGER*4 ) -- Which model for the Love numbers of *
! *                                  the third degree is to be applied. *
! *                               SOTID__3D_NONE                       *
```

```
! *                              SOTID__3D_MDG97                        *
! *                                                                     *
! *   Named constants SOTID__xxx are defined in sotid_data.i            *
! *   Their descriptipon can be found in documentation for SOTID.       *
! *                                                                     *
! * ________________________ Output parameters: _______________________ *
! *                                                                     *
! *         TIDCNF ( RECORD    ) -- Object which holds configuration    *
! *                                   parameters of SOTID.              *
! * ________________________ Modified parameters: _____________________ *
! *                                                                     *
! *    IUER ( INTEGER*4      ) -- Universal error handler.              *
! *                             Input: switch IUER=0 -- no error messages  *
! *                                    will be generated even in the case  *
! *                                    of error. IUER=-1 -- in the case of *
! *                                    error the message will be put in    *
! *                                    stdout.                          *
! *                             Output: 0 in the case of successful     *
! *                                     completion and non-zero in the  *
! *                                     case of error.                  *
```

2.2 SOTID_PRE

```
! *************************************************************************
! *                                                                     *
! *     Subroutine  SOTID_PRE  computes time-independent arguments      *
! *   used for computations of site displacements caused by solid Earth  *
! *   tides for the set of stations. These arguments depends on         *
! *   crust-fixed station positions. Routine SOTDID_PRE puts these      *
! *   time-independent arguments into the STATID arrays.                *
! *                                                                     *
! * _________________________ Input parameters: _______________________ *
! *                                                                     *
! *  L_STA ( INTEGER*4 ) -- Total number of stations.                   *
! *  C_STA ( CHARACTER ) -- Array of station names. Dimension: L_STA.   *
! *  R_CFS ( REAL*8    ) -- Array of crust-fixed station positions.     *
! *                         Dimension: 3,L_STA. Units: meters.          *
! *                                                                     *
! * _________________________ Output parameters: ______________________ *
! *                                                                     *
! * TIDCNF ( RECORD    ) -- object which holds configuration parameters  *
! *                         of SOTID.                                   *
! * STATID ( RECORD    ) -- object with information about station.      *
! *                         SOTID_PRE fills some fields of this object.  *
! *    IUER ( INTEGER*4      ) -- Universal error handler.              *
! *                             Input: switch IUER=0 -- no error messages  *
! *                                    will be generated even in the case  *
! *                                    of error. IUER=-1 -- in the case of *
```

```
! *                                   error the message will be put in    *
! *                                   stdout.                             *
! *                            Output: 0 in the case of successful        *
! *                                    completion and non-zero in the     *
! *                                    case of error.                     *
```

2.3 SOTID_TIM

```
! ************************************************************************
! *                                                                       *
! *   Routine  SOTID_TIM  calculates intermediary time-dependent sums    *
! *   over harmonics of the tidal potential of the second and the third  *
! *   degree. These quantities are used in calculation displacements     *
! *   due to Earth solid tides, pole tides and ocean loading. They are   *
! *   put in the object TIDCNF. Routine SOTID_TIM should be called each  *
! *   time for the new moment of time before SOTID_DSP which computes    *
! *   tidal displacement for the specific moment of time specified in    *
! *   the call to SOTID_TIM for the specific station.                    *
! *   Truncated harmonic expansion HW95 is used for computation.         *
! *                                                                       *
! * ________________________ Input parameters:  ________________________ *
! *                                                                       *
! *       MJD ( INTEGER*4 ) -- Integer fraction of the Modified Julian   *
! *                            Day -- MJD at the midnight of the         *
! *                            observations. It has the meaning of the   *
! *                            INTEGER number of days elapsed from       *
! *                            0 hours of 01 January 2000).              *
! *       TAI ( REAL*8    ) -- Time at TAI scale of the moment under     *
! *                            consideration (in sec).                   *
! * UT1_M_TAI ( REAL*8    ) -- Value of the function UT1-TAI at the      *
! *                            moment of observation. This quantity is   *
! *                            obtained from the analysis of the         *
! *                            astronomical observations.                *
! *    TIDCNF ( RECORD    ) -- Object which holds configuration          *
! *                            parameters of SOTID.                      *
! *                                                                       *
! * ________________________ Output parameters: ________________________ *
! *                                                                       *
! *    TIMTID ( RECORD    ) -- Object with time-dependent intermediary   *
! *                            quantities used for computation of        *
! *                            displacements caused by the Earth's solid *
! *                            tides.                                    *
! *                                                                       *
! * ________________________ Modified parameters: ______________________ *
! *                                                                       *
! *    IUER ( INTEGER*4      ) -- Universal error handler.               *
! *                            Input: switch IUER=0 -- no error messages *
! *                                   will be generated even in the case *
```

```
! *                                  of error. IUER=-1 -- in the case of *
! *                                  error the message will be put in    *
! *                                  stdout.                             *
! *                           Output: 0 in the case of successful        *
! *                                   completion and non-zero in the     *
! *                                   case of error.                     *
```

2.4 SOTID_DSP

```
! ************************************************************************
! *                                                                      *
! *     Routine  SOTID_DSP  calculates site displacements due to the     *
! *   Earth's tides of the second order and third order using harmonic   *
! *   expansion of tidal potential according to configuration parameters *
! *   of SOTID. It is assumed that session-dependent and time-dependent  *
! *   intermediary quantities were calculated before:                    *
! *                                                                      *
! *   CALL SOTID_SET ( ... ) -- set SOTID configuration. Called once     *
! *                             in a program.                            *
! *   CALL SOTID_PRE ( ... ) -- compute station specific,                *
! *                             time-independent intermediary variables. *
! *                             Called once in a program.                *
! *   CALL SOTID_TIM ( ... ) -- compute station-independent,             *
! *                             time-specific intermediary variables on  *
! *                             the moment of time under consideration.  *
! *                                                                      *
! *   Thus, SOTID_DSP computes site displacement for the station         *
! *   specified in the call SOTID_PRE for the moment of time specified   *
! *   in the call SOTID_TIM according to configuration specified in      *
! *   the call SOTID_SET.                                                *
! *                                                                      *
! * ________________________ Input parameters:  ________________________ *
! *                                                                      *
! *   TIDCNF ( RECORD    ) -- Object which holds configuration           *
! *                           parameters of SOTID.                       *
! *   TIMTID ( RECORD    ) -- Object with time-dependent intermediary    *
! *                           quantities used fro computation of         *
! *                           displacements caused by the Earth's solid  *
! *                           tides.                                     *
! *   STATID ( RECORD    ) -- Object with information about station.     *
! *                           SOTID_PRE fills some fields of this object.*
! *                                                                      *
! * ________________________ Output parameters: ________________________ *
! *                                                                      *
! *    D_REN ( REAL*8    ) -- Vector of tidal displacement in REN        *
! *                           topocentric system: radial(up), east,      *
! *                           north. Units: meters.                      *
! *                                                                      *
```

2.5 SOTID_INQ

```
! **********************************************************************
! *                                                                    *
! *   Function SOTID_INQ  makes an inquiry of the current SOTID        *
! *   configuration and returns the status. One inquiry at the time    *
! *   can be made. SOTID_INQ returns the value of the requested        *
! *   parameter as well as the output string with parameter description. *
! *                                                                    *
! * ________________________ Input parameters:  ________________________ *
! *                                                                    *
! *     REQUEST ( INTEGER*4 ) -- Code of the request. The following    *
! *                              symbolic names for a request are      *
! *                              supported:                            *
! *               SOTID__REQ_MODEL_2D -- what is the model for         *
! *                                      computation of the tides of the *
! *                                      second degree.                *
! *               SOTID__REQ_MODEL_3D -- what is the model for         *
! *                                      computation of the tides of the *
! *                                      third degree.                 *
! *               SOTID__REQ_GEN_LOVE -- which generalized Love numbers *
! *                                      to take into account.         *
! *               SOTID__REQ_ORDER_2D -- waves of which order, degree 2 *
! *                                      to take into account.         *
! *               SOTID__REQ_ZF_LOVE  -- what is the model for Love    *
! *                                      numbers for the zero frequency. *
! *               SOTID__REQ_N_STA    -- for how many stations STATID  *
! *                                      record was set up.            *
! *               SOTID__REQ_NW_D2    -- how many tidal waves of the   *
! *                                      potential degree 2 are used.  *
! *               SOTID__REQ_NW_D3    -- how many tidal waves of the   *
! *                                      potential degree 3 are used.  *
! *               If the code of thre request is not the one from the  *
! *               list above, then SOTID_INQ will return the answer    *
! *               SOTID__UNDEFINED, "undefined".                       *
! *                                                                    *
! *      TIDCNF ( RECORD    ) -- Object which holds configuration      *
! *                              parameters of SOTID.                  *
! *                                                                    *
! * ________________________ Output parameters: ________________________ *
! *                                                                    *
! *     OUT_STR ( CHARACTER ) -- Descriptive string of the answer to the *
! *                              request.                              *
! * OUT_STR_LEN ( CHARACTER ) -- The effective length of OUT_STR       *
! *                              (position of the last character which *
! *                               is not a blank).                     *
! *                                                                    *
! * ________________________ Modified parameters: ______________________ *
```

```
! *                                                                  *
! *    IUER ( INTEGER*4      ) -- Universal error handler.           *
! *                            Input: switch IUER=0 -- no error messages  *
! *                                   will be generated even in the case  *
! *                                   of error. IUER=-1 -- in the case of *
! *                                   error the message will be put in    *
! *                                   stdout.                             *
! *                            Output: 0 in the case of successful        *
! *                                    completion and non-zero in the     *
! *                                    case of error.                     *
```

3 Configuration

Meaning of configuration parameters:

- GEN_LOVE — Which generalized Love numbers are to be used. Sotid allows us to restrict generalized Love numbers of the second degree if it seems desirable. This option is applied to the second degree Love numbers only. Supported values:
  - SOTID__PRN_ONLY only principal Love numbers, h_2, ℓ_2, will be used;
  - SOTID__PL_ONLY only principal and latitude-dependent Love numbers will be used.
  - SOTID__GEN_ALL all generalized Love numbers of the second degree will be used.
- MODEL_2D – Which model for the Love numbers of the second degree is to be applied. Sotid supports a collection of Love numbers models.
  - SOTID__LOVE Frequency independent model for Love numbers [8]. This model has all Love numbers, except principal, equal to zero.
  - SOTID__IERS92 Frequency independent model for Love numbers [6], except the h_2 Love number at the K_1 frequency ($7.292115855138 \cdot 10^-5$ rad/sec). This model has all Love numbers, except principal, equal to zero.
  - SOTID__MDG97EL Elastic variant of the MDG97 [4] model.
  - SOTID__MDG97AN Anelastic variant of the MDG97 [4] model.
  - SOTID__DDW99EH Elastic, hydrostatic variant of the DDW99 [1] model.
  - SOTID__DDW99IN Inelastic, non-hydrostatic variant of the DDW99 [1] model.
  - SOTID__MAT00 Model of P. Mathews submitted to IERS Conventions, 2000. Revision of 2001.10.10. [7]
  - SOTID__MAT01 Model of P. Mathews 2001 [5]
- ORDER_2D – Tidal waves of which order of the second degree should be taken into account: zonal, diurnal, semi-diurnal or their combination. Sotid allows to specify any combination of orders, but this feature is reserved rather for testing and special use. All orders should used in normal mode of calculations.
  - SOTID__2D_012ORD — all orders will be used. This option should be normally used.
  - SOTID__2D_NONE — no tides of the second order will be taken into account.
  - SOTID__2D_0ORD — only zonal tides are taken into account.

  - SOTID__2D_01ORD — only zonal and diurnal tides are taken into account.
  - SOTID__2D_02ORD — only zonal and semi-diurnal tides are taken into account.
  - SOTID__2D_1ORD — only diurnal tides are taken into account.
  - SOTID__2D_12ORD — only diurnal and semi-diurnal tides are taken into account.
  - SOTID__2D_2ORD — only semi-diurnal tides are taken into account.

- ZERO_FREQ_LOVE Which Love numbers for the tides of the zero frequency should be used. Displacement with the zero frequency means a permanent tide. Since permanent tide cannot be distinguished from site position, the question of which Love numbers should be assign for the zero frequency is the question of an agreement. Although the 18-th IAG General Assembly approved the resolution 16 advocated SOTID__ZF_FLUID model, SOTID__ZF_MDG97EL or SOTID__ZF_MDG97AN were widely used in practice in 1990s.

  - SOTID__ZF_ZERO All Love numbers for zeroth frequency are zero. Therefore, tides are defined as displacements with zero mean.
  - SOTID__ZF_LOVE Love numbers for the zeroth frequency are the same as in SOTID__LOVE model: they do not depend on frequency.
  - SOTID__ZF_MDG97EL Love numbers for zero frequency are defined in accordance with MDG97 model, elastic variant. Love numbers for the zero frequency are the same as for semi-diurnal tides.
  - SOTID__ZF_MDG97AN Love numbers for zero frequency are defined in accordance with MDG97 model, anelastic variant. Love numbers for the zero frequency are the same as for semi-diurnal tides.
  - SOTID__ZF_FLUID Love numbers for zero frequency are determined by fluid limit. By another words, what would be a displacement if the Earth's crust were fluid.

  NB: there is no agreement how to interpret the phrase to "apply" or "remove" permanent tide deformation. In the framework of the proposed approach permanent deformation is computed in accordance with the assigned Love numbers for the zero frequency. According to the approach outlined in IERS Conventions [7] proposed in [4], displacements first computed using frequency independent Love numbers for all constituents, including the zero frequency tide, them later they are "corrected" for frequency dependence of Love numbers. Therefore, "applying" or "removal" of permanent tide deformation is reduced to the choice of Love numbers for the zero frequency.

- MODEL_3D Which model for the Love numbers of the third degree is to be applied. Sotid supports only one model for tides of the third degree.

  - SOTID__3D_NONE — not to take into account tides of the third degree.
  - SOTID__3D_MDG97 — MDG97 model [4] should be applied for computation of tides of the third degree. This model proposes frequency-independent Love numbers h_3 and ℓ_3.

4 Algorithm

Appendix A of paper [9] gives the following expression for site displacements due to the solid Earth's tides of the second degree:

$$
\begin{aligned}
\vec{d}_{\mathsf{ren}} = \\
\sum_{m=0}^{m=2} & \begin{pmatrix}
\vec{X}_1^{rc}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_1^r(k)\, \cos\gamma_{km} & - & \vec{X}_1^{rs}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_1^r(k)\, \sin\gamma_{km} \\
\vec{X}_2^{rc}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_2^r(k)\, \sin\gamma_{km} & + & \vec{X}_2^{rs}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k \vec{L}_2^r(k)\, \cos\gamma_{km} \\
\vec{X}_3^{rc}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_3^r(k)\, \cos\gamma_{km} & - & \vec{X}_3^{rs}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_3^r(k)\, \sin\gamma_{km}
\end{pmatrix} + \\
\sum_{m=0}^{m=2} & \begin{pmatrix}
-\vec{X}_1^{ic}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_1^i(k)\, \sin\gamma_{km} & - & \vec{X}_1^{is}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_1^i(k)\, \cos\gamma_{km} \\
\vec{X}_2^{ic}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_2^i(k)\, \cos\gamma_{km} & - & \vec{X}_2^{is}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_2^i(k)\, \sin\gamma_{km} \\
-\vec{X}_3^{ic}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_3^i(k)\, \sin\gamma_{km} & - & \vec{X}_3^{is}(m,\varphi) \cdot \sum\limits_{k=1}^{n(m)} A_k\, \vec{L}_3^i(k)\, \cos\gamma_{km}
\end{pmatrix}
\end{aligned}
\tag{1}
$$

where vector $\vec{X}$ depends only on station coordinates:

$$
\begin{array}{ll}
\vec{X}_j^{rc}(m,\varphi) = \vec{Z}_j^r(m,\varphi) \cdot \cos m\lambda & \qquad \vec{X}_j^{rs}(m,\varphi) = \vec{Z}_j^r(m,\varphi) \cdot \sin m\lambda \\
\vec{X}_j^{ic}(m,\varphi) = \vec{Z}_j^i(m,\varphi) \cdot \cos m\lambda & \qquad \vec{X}_j^{is}(m,\varphi) = \vec{Z}_j^i(m,\varphi) \cdot \sin m\lambda
\end{array}
$$

where φ is a geocentric latitude and λ is a positive towards east longitude. Vector $\vec{Z}$ is

$$
\begin{array}{lll}
\vec{Z}_1^r = \begin{pmatrix} \bar{P}_2^m \dfrac{1}{g_e} \\ P_2^0\, \bar{P}_2^m \dfrac{1}{g_e} \\ 0 \\ \dfrac{1}{g_e} \end{pmatrix} &
\vec{Z}_2^r = \begin{pmatrix} -\dfrac{m}{\cos\varphi}\, \bar{P}_2^m \dfrac{1}{g_e} \\ -\dfrac{m}{\cos\varphi}\, P_2^0\, \bar{P}_2^m \dfrac{1}{g_e} \\ P_1^0 \dfrac{\partial \bar{P}_2^m}{\partial\varphi} \dfrac{1}{g_e} \\ \dfrac{\partial \bar{P}_1^m}{\partial\varphi} \dfrac{1}{g_e} \end{pmatrix} &
\vec{Z}_3^r = \begin{pmatrix} \dfrac{\partial \bar{P}_2^m}{\partial\varphi} \dfrac{1}{g_e} \\ P_2^0 \dfrac{\partial \bar{P}_2^m}{\partial\varphi} \dfrac{1}{g_e} \\ -\dfrac{m}{\cos\varphi}\, P_1^0\, \bar{P}_2^m \dfrac{1}{g_e} \\ -\dfrac{m}{\cos\varphi}\, \bar{P}_1^m \dfrac{1}{g_e} \end{pmatrix} \\[2ex]
\vec{Z}_1^i = \begin{pmatrix} \bar{P}_2^m \dfrac{1}{g_e} \end{pmatrix} &
\vec{Z}_2^i = \begin{pmatrix} -\dfrac{m}{\cos\varphi}\, \bar{P}_2^m \dfrac{1}{g_e} \end{pmatrix} &
\vec{Z}_3^i = \begin{pmatrix} \dfrac{\partial \bar{P}_2^m}{\partial\varphi} \dfrac{1}{g_e} \end{pmatrix}
\end{array}
\tag{2}
$$

P_m^0 is a Legendre function normalized to have maximal value 1:

$$
\begin{array}{lll}
P_1^0 = \sin\varphi & P_1^1 = \cos\varphi & P_1^2 = 0 \\
P_2^0 = \left(\dfrac{3}{2}\sin^2\varphi - \dfrac{1}{2}\right) & P_2^1 = 2\sin\varphi\,\cos\varphi & P_2^2 = \cos^2\varphi
\end{array}
\tag{3}
$$

and $\bar{P}_l^m$ are Legendre functions normalized over the surface of the unit sphere:

$$
\begin{array}{lll}
\bar{P}_1^0 = P_1^0 & \bar{P}_1^1 = P_1^1 & \bar{P}_1^2 = P_1^2 \\
\bar{P}_2^0 = \sqrt{\dfrac{5}{4\pi}}\,P_2^0 & \bar{P}_2^1 = \sqrt{\dfrac{15}{32\pi}}\,P_2^1 & \bar{P}_2^2 = \sqrt{\dfrac{15}{32\pi}}\,P_2^2
\end{array}
\tag{4}
$$

g_e — the equatorial Earth's gravity acceleration.

The summing in (1) is done over the constituents of the spectral expansion of the tide-generating potential which is assumed to be in the form

$$
\Phi_2^m(t,r) = \sum_{m=0}^{m=2} \left(\frac{r}{a}\right)^2 \bar{P}_2^m(\varphi) \cdot \sum_k^{n(m)} A_k \cdot \cos\gamma_{km}
\tag{5}
$$

where r — is the distance from the geocenter, a — is the semi-major axis of the Earth, A_k is the normalized amplitude of the k-th tidal wave and γ_{km} is its argument:

$$
\gamma_{km} = \psi_k + \theta_k + \omega_k\, t_{\mathsf{tdb}} + m\,\frac{2\pi(\mathsf{ut1} - t_{\mathsf{tdb}})}{86400}
\tag{6}
$$

ψ_k is the phase of the k-th wave, θ_k and ω_k are the phase and frequency of the harmonic argument of that wave. t_{tdb} is the time elapsed since the fundamental epoch J2000.0 (12^h 1 January, 2000) at the TDB scale. The difference $\mathsf{ut1} - t_{\mathsf{tdb}}$ in (6) takes into account variations in the Earth's rotation which were omitted in producing the tidal potential series. The variable m=0,1,2 denotes the order of a tidal wave, subscript index 1,2,3 denotes component of the displacement vector: radial, east, north, and summation is carried out over spectral harmonics of the tidal expansion.

The vector of generalized Love numbers is presented in the form

$$
\begin{array}{lllclll}
\vec{L}_1^r & = & (\,h^{(0)},\, h^{(2)},\, 0,\, h'\,)^\top & \qquad & \vec{L}_1^i & = & h^{(i)} \\
\vec{L}_2^r & = & (\,l^{(0)},\, l^{(2)},\, l^{(1)},\, l'\,)^\top & & \vec{L}_2^i & = & l^{(i)} \\
\vec{L}_3^r & = & (\,l^{(0)},\, l^{(2)},\, l^{(1)},\, l')^\top & & \vec{L}_3^i & = & l^{(i)}
\end{array}
\tag{7}
$$

where

$h^{(0)}$ — principal Love number;
$h^{(i)}$ — out-of-phase radial Love number;
$h^{(2)}$ — latitude Love number ;
h' — zero degree Love number;
$l^{(0)}$ — principal Shida number;
$l^{(i)}$ — out-of-phase Shida number;
$l^{(1)}$ — second degree toroidal Love number;
$l^{(2)}$ — latitude Shida number;
l' — first degree toroidal Love number;

Appendix B of paper [9] gives the following expression for site displacements due to the solid Earth's tides of the third degree:

$$\vec{d}_{\mathsf{ren}} = \sum_{m=0}^{m=3} \begin{pmatrix} X_1^{3c}(m,\varphi)\, h_3 \cdot \sum\limits_{k=1}^{n(m)} A_k \, \cos\gamma_{km} & - & X_1^{3s}(m,\varphi)\, h_3 \cdot \sum\limits_{k=1}^{n(m)} A_k \, \sin\gamma_{km} \\ X_2^{3s}(m,\varphi)\, l_3 \cdot \sum\limits_{k=1}^{n(m)} A_k \, \sin\gamma_{km} & + & X_2^{3c}(m,\varphi)\, l_3 \cdot \sum\limits_{k=1}^{n(m)} A_k \, \cos\gamma_{km} \\ X_3^{3c}(m,\varphi)\, l_3 \cdot \sum\limits_{k=1}^{n(m)} A_k \, \cos\gamma_{km} & - & X_3^{3s}(m,\varphi)\, l_3 \cdot \sum\limits_{k=1}^{n(m)} A_k \, \sin\gamma_{km} \end{pmatrix} \tag{8}$$

where X depends only on station coordinates

$$X_j^{3c}(m,\varphi) = Z_j^3(m,\varphi) \cdot \cos m\lambda \qquad X_j^{3s}(m,\varphi) = Z_j^3(m,\varphi) \cdot \sin m\lambda$$

and Z_j^3 is

$$Z_1^3(m,\varphi) = \bar{P}_3^m \frac{1}{g_e} \qquad Z_2^3(m,\varphi) = -\frac{m}{\cos\varphi} \bar{P}_3^m \frac{1}{g_e} \qquad Z_3^3(m,\varphi) = \frac{\partial \bar{P}_3^m}{\partial\varphi} \frac{1}{g_e} \tag{9}$$

Legendre functions of third order are

$$\begin{array}{ll} P_3^0 = \left(\frac{5}{2}\sin^3\varphi - \frac{3}{2}\sin\varphi\right) & P_3^1 = \left(\frac{5}{2}\sin^2\varphi - \frac{1}{2}\sin\varphi\right)\cos\varphi \\ P_3^2 = \sin\varphi\cos^2\varphi & P_3^3 = \cos^3\varphi \end{array} \tag{10}$$

$$\bar{P}_3^0 = \sqrt{\frac{7}{4\,\pi}} P_3^0 \qquad \bar{P}_3^1 = \sqrt{\frac{21}{16\,\pi}} P_3^1 \qquad \bar{P}_3^2 = \sqrt{\frac{105}{32\,\pi}} P_3^2 \qquad \bar{P}_3^3 = \sqrt{\frac{35}{64\,\pi}} P_3^3 \tag{11}$$

Computation is done in several steps. First, HW95 expansion of the tide-generating potential [2] is transformed to a pure harmonic form. The expansion contains terms C_0, S_0, C_1, S_1 and the frequency of every tidal wave is implicitly defined through the sum of fundamental arguments. Program hw95_out transforms HW95 catalogue to another form:

$$\psi_k = -\,\mathrm{arctan}\, \frac{S_{0k}}{C_{0k}}$$

$$A_k^m = \rho(m)\,\sqrt{C_{0k}^2 + S_{0k}^2} \tag{12}$$

$$\tag{13}$$

$\rho(m)$ — is a re-normalization factor. It is $\sqrt{4\pi}$ for tides of the 0-th order and $\sqrt{8\pi}$ for other tides.

Frequencies and phases of tidal constituents are easily computed via coefficients at fundamental arguments.

$$\begin{aligned} \theta_i &= \sum_{j=1}^{j=11} k_{ij}\, F_{jo} + \theta_{ai} \\ \omega_i &= \sum_{j=1}^{j=11} k_{ij}\, F_{j1} + \omega_{ai} \end{aligned} \tag{14}$$

where F_{jq} are fundamental coefficients form the theory of planet motion [10]. Here we neglected terms of the 2-nd degree and higher.

Program hw95_out processes all tidal constituents of the second and third degree with the amplitudes greater than a specified threshold. It combines synonymous tidal waves with the same arguments and sorts tidal constituents in decreasing order of their amplitudes. The output of hw95_out program is formatted in the Fortran90 DATA statements format. This output is used as constants in sotid routines. Sotid is compiled with hw95_2d_0002.i and hw95_3d_0002.i include files for tide generating potential of the 2-nd and 3-rd order respectively. Truncation level is 0.0002 m^2/s^2 is chosen what is sufficient for computation of the displacements with precision of better than 0.05 mm — much better than the accuracy of tidal theory. In the case if a user would like to use another truncation level, the program hw95_out can be used for that. Then filenames with results of hs95_out should replace last two lines sotid_data.i file abd sotid should be re-compiled.

The second step is to compute $\vec{X}^{ab}(m,\varphi)$ vectors. These quantities depend on site position, but does not depend on time. Program sotid_pre makes these computations.

The third step is to compute sums $\sum\limits_{k=1}^{n(m)} A_k \vec{L}_{jk} \cos\gamma_{km}$ and $\sum\limits_{k=1}^{n(m)} A_k \vec{L}_{jk} \sin\gamma_{km}$. These quantities depends only on time, but do not depend on site position. Computation of the generalized Love numbers which enters in $\vec{L}_{jk}$ depends on the model. SOTID_LOVE model considers Love numbers as constants. SOTID_IERS92 model considers Love numbers as constants, except the h_2 Love number at the K_1 frequency ($h_2(K_1) = 0.5203$). SOTID_MDG97EL, SOTID_MDG97AN, SOTID_DDW99EH, SOTID_DDW99AN models considers Love numbers for semi-diurnal tides as constants. Love numbers for diurnal tides are approximated by the expression

$$\mathcal{L}(f) = d_1 + d_2 f^3 + \frac{d_3}{f_o - f} \tag{15}$$

where f is a frequency. Parameters d_1, d_2, d_3, f_o are obtained by the LSQ fits from the tables.

Love numbers for zonal tides are approximated by the expression

$$\mathcal{L}(f) = z_1 \log(f) + z_2 \log^2(f) \tag{16}$$

Complex Love numbers in accordance with SOTID_MAT00 and SOTID_MAT01 models for diurnal band are computed in accordance with the generic formula [3] as

$$\mathcal{L}(f) = L_o + \sum_{s=1}^{s=3} \frac{L_s}{f - f_s} \tag{17}$$

where the complex coefficients L_s, and complex frequencies f_s are taken from the original papers [5], [7]. In addition to this resonance formulas, generalized Love numbers for the set of frequencies in diurnal band, 24 in SOTID_MAT00 model, and 16 in the SOTID_MAT01, are taken from the tables presented in the original papers. The values from the tables replace the values computed in accordance with the resonance formula in order to compensate inaccuracy of the generic formula.

Love numbers for zonal tides in accordance with SOTID_MAT00 and SOTID_MAT01 models are computed with using the following expression [7]:

$$L_o = L_{zo} + L_{z1} \left[\cot\left(\frac{\alpha\pi}{2}\right) \left[1 - \left(\frac{f_m}{f}\right)^{\alpha} \right] \; + \; i \left(\frac{f_m}{f}\right)^{\alpha} \right] \tag{18}$$

Models SOTID_MAT00 and SOTID_MAT01 have the same coefficients for Love numbers for zonal tides.

The last, fourth step is to compute displacements using expressions 1 and 8.

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