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MEMO: ionosphere contribtuion in VLBI

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Abstract:

Expression for ionosphere contribution is given. The way how ionosphere contribution is handled in Solve is described.

Introduction

Ionosphere is the layer of plasma surrounding the Earth at heights 70–300 km. Electron density varies by two orders of magnitudes on time scales from several minutes due ionosphere scintillation to decades due to Solar activity. Currently, there are no methods for prediction of electron density of ionosphere. However, ionosphere index of refractivity has strong dependency on frequency. Therefore, observing at two widely separated frequencies allows us to determine ionosphere contribution and to form a combination of observables which is free from ionosphere contribution. Theoretical expression for ionosphere contribution to phase and group delay obtained by band synthesis is given in chapters 1–2. Implementation of this technique in Solve is discussed in chapter 3.

1 Theoretical expression for ionosphere contribution to delays

I follow here treatment of ionosphere presented in T. Herring PhD theses. Electromagnetic wave propagates in plasma with phase velocity

$$v_p = c\sqrt{1 - \frac{N_v\, e^2}{m_e\, \varepsilon_o\, \omega^2}} \tag{1}$$

where N_v — electron density, e — charge of an electron, m_e — mass of an electron, ε_o — permittivity of free space, ω — cyclic frequency of the wave and c — velocity of light in vacuum. Phase velocity in ionosphere is faster than velocity of light in vacuum.

After integration along ray path, expanding expression 1 with holding only the term of the first order, we get the following expression for additional phase rotation caused by ionosphere:

$$\Delta\varphi = -\frac{\alpha}{\omega} \tag{2}$$

where α is

$$\alpha = \frac{e^2}{2\, c\, m_e\, \varepsilon_o} \left(\int N_v\, ds_1 - \int N_v\, ds_2 \right) \tag{3}$$

s_1 and s_2 are paths of wave propagation from a source to the first and second station of the radio interferometer.

If to exprss $\int N_v\, ds$ in units $1 \cdot 10^{16}$ electrons$/m^2$ (so-called TEC units) then after having substituted values of constants we get

$$\alpha = 3.403 \cdot 10^{-11}\ \sec^{-1}$$

A fringe phase in channel i is expressed as

$$\varphi_i = \tau_{ph}\, \omega_o + \tau_{gr}\, (\omega_i - \omega_o) - \frac{\alpha}{\omega_i} \tag{4}$$

where ω_o — is a reference sky frequency.

Unknown quantities phase and group delays: τ_{ph} and τ_{gr} can be determined from equations (4) by using weighted LSQ. Equations of conditions for such a problem can be written as

$$\tau_{ph} \frac{\partial \varphi_i}{\partial \tau_{ph}} + \tau_{gr} \frac{\partial \varphi_i}{\partial \tau_{gr}} = \varphi_i + \frac{\alpha}{\omega_i} \tag{5}$$

Actually, group and phase delays are obtained in fringing software by minimizing delay resolution function. However, it can be demonstrated that this method gives the same results as solving equations (5) by LSQ provided

- weights $\dfrac{(U_i + L_i)\, A_i}{\nu}$ are ascribed equations of conditions. U_i and L_i is the number of processed samples in upper and lower sideband of the i-th channel, A_i amplitude in this channel, ν — sampling rate.
- residual phases are not large. In practice the difference in estimates of group delay obtained by minimizing delay resolution function and by solving equations of conditions does not exceed 0.1 psec if residual phases are less than 0.4 rad (23°).

Now let's obtain explicit expression for delays by solving (4) by LSQ:

$$\hat{x} = \left((r A^{\top})\, r A \right)^{-1}\, (r A^{\top})\, x \tag{6}$$

where A — matrix of equations of conditions, r — vector of weights, x — vector of right parts of equations of conditions.

Partial derivatives are

$$\frac{\partial \varphi_i}{\partial \tau_{ph}} = \omega_o \qquad\qquad \frac{\partial \varphi_i}{\partial \tau_{gr}} = \omega_i - \omega_o \tag{7}$$

We can use the explicit expression for an invert of a symmetrical 2x2 matrix:

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{pmatrix}^{-1} = \begin{pmatrix} \dfrac{A_{22}}{\Delta} & -\dfrac{A_{12}}{\Delta} \\ -\dfrac{A_{12}}{\Delta} & \dfrac{A_{11}}{\Delta} \end{pmatrix} \qquad \Delta = A_{11}A_{22} - A_{12}^2 \tag{8}$$

We can easily find blocks of normal matrix and normal vector:

$$\begin{aligned} rA_{11} &= \omega_o^2 \sum_i^n r_i \\ rA_{12} &= \omega_o \sum_i^n r_i(\omega_i - \omega_o) \\ rA_{22} &= \sum_i^n r_i(\omega_i - \omega_o)^2 \\ \Delta &= \omega_o^2 \left(\sum_i^n r_i \cdot \sum_i^n r_i(\omega_i - \omega_o)^2 - \left(\sum_i^n r_i(\omega_i - \omega_o) \right)^2 \right) \end{aligned} \tag{9}$$

Then we can write solution of system of normal equations (7):

$$\begin{pmatrix} \tau_{ph} \\ \tau_{gr} \end{pmatrix} = \begin{pmatrix} \dfrac{A_{22}}{\Delta} & -\dfrac{A_{12}}{\Delta} \\ -\dfrac{A_{12}}{\Delta} & \dfrac{A_{11}}{\Delta} \end{pmatrix} \begin{pmatrix} \omega_o \sum\limits_i^n r_i \varphi_i & + \alpha\, \omega_o \sum\limits_i^n \dfrac{r_i}{\omega_i} \\ \omega_o \sum\limits_i^n r_i(\omega_i - \omega_o)\varphi_i + \alpha & \sum\limits_i^n r_i \dfrac{\omega_i - \omega_o}{\omega_i} \end{pmatrix} \tag{10}$$

or after some algebra

$$\begin{aligned} \tau_{ph} &= \frac{\sum\limits_i^n r_i(\omega_i - \omega_o)^2 \cdot \sum\limits_i^n r_i\varphi_i - \sum\limits_i^n r_i(\omega_i - \omega_o) \sum\limits_i^n r_i(\omega_i - \omega_o)\varphi_i}{\omega_o \left(\sum\limits_i^n r_i \cdot \sum\limits_i^n r_i(\omega_i - \omega_o)^2 - \left(\sum\limits_i^n r_i(\omega_i - \omega_o) \right)^2 \right)} - \frac{\alpha}{\omega_{ph}^2} \\ \tau_{gr} &= \frac{\sum\limits_i^n r_i \cdot \sum\limits_i^n r_i(\omega_i - \omega_o)\varphi_i - \sum\limits_i^n r_i(\omega_i - \omega_o)^2 \sum\limits_i^n r_i\varphi_i}{\left(\sum\limits_i^n r_i \cdot \sum\limits_i^n r_i(\omega_i - \omega_o)^2 - \left(\sum\limits_i^n r_i(\omega_i - \omega_o) \right)^2 \right)} + \frac{\alpha}{\omega_{gr}^2} \end{aligned} \tag{11}$$

where ω_{ph} and ω_{gr} are

$$\omega_{ph} = \sqrt{\omega_o \frac{\sum\limits_i^n r_i \cdot \sum\limits_i^n r_i(\omega_i - \omega_o)^2 - \left(\sum\limits_i^n r_i(\omega_i - \omega_o)\right)^2}{\sum\limits_i^n r_i(\omega_i - \omega_o) \sum\limits_i^n r_i \frac{(\omega_i - \omega_o)}{\omega_i} - \sum\limits_i^n r_i(\omega_i - \omega_o)^2 \cdot \sum\limits_i^n \frac{r_i}{\omega_i}}}$$
$$\omega_{gr} = \sqrt{\frac{\sum\limits_i^n r_i \cdot \sum\limits_i^n r_i(\omega_i - \omega_o)^2 - \left(\sum\limits_i^n r_i(\omega_i - \omega_o)\right)^2}{\sum\limits_i^n r_i(\omega_i - \omega_o)^2 \sum\limits_i^n \frac{r_i}{\omega_i} - \sum\limits_i^n r_i \cdot \sum\limits_i^n r_i \frac{(\omega_i - \omega_o)}{\omega_i}}} \qquad (12)$$

ω_{ph} and ω_{gr} are called effective ionosphere frequencies for ionosphere contribution. They have clear physical meaning: if the wideband signal be replaced by a quasi-monochromatic signal with a group or phase effective ionosphere frequency then contribution to group or phase delay would be the same.

2 Ionosphere-free linear combinations

Using notion of ionosphere effective frequencies we can express observed group and phase delays at X and S bands through ionosphere free delay τ_{if} and parameter α:

$$\begin{aligned}
\tau_{gx} &= \tau_{if} + \frac{\alpha}{\omega_{gx}^2} \\
\tau_{gs} &= \tau_{if} + \frac{\alpha}{\omega_{gs}^2} \\
\tau_{px} &= \tau_{if} - \frac{\alpha}{\omega_{px}^2} \\
\tau_{ps} &= \tau_{if} - \frac{\alpha}{\omega_{ps}^2}
\end{aligned} \qquad (13)$$

Here the first letter in indices stands for group or phase delay and the second letter stands for X or S band. Using these equations we can eliminate unknown parameter α and express ionosphere free delay through a linear combination of two or three observables. The most important ionosphere-free linear combination of observables are given below:

$$\begin{aligned}
\text{G_Gxs} &= \frac{\omega_{gx}^2}{\omega_{gx}^2 - \omega_{gs}^2}\tau_{gx} - \frac{\omega_{gs}^2}{\omega_{gx}^2 - \omega_{gs}^2}\tau_{gs} \\
\text{PxGs} &= \frac{\omega_{px}^2}{\omega_{px}^2 + \omega_{gs}^2}\tau_{px} + \frac{\omega_{gs}^2}{\omega_{px}^2 + \omega_{gs}^2}\tau_{gs} \\
\text{PxGx} &= \frac{\omega_{px}^2}{\omega_{px}^2 + \omega_{gx}^2}\tau_{px} + \frac{\omega_{gx}^2}{\omega_{px}^2 + \omega_{gx}^2}\tau_{gx}
\end{aligned} \qquad (14)$$

Alternative way is to use expression for ionosphere contribution to group delay at X band:

$$\tau_{igx} = -\frac{\omega_{gs}^2}{\omega_{gx}^2 - \omega_{gs}^2} (\tau_{gx} - \tau_{gs}) \tag{15}$$

Then we can add it to theoretical delay and thus "correct" or "calibrate" group delay at X band for ionosphere contribution. This approach is rather ugly since "calibration" or "correction" to group delay contains this quantity (We correct observable X using the measurement of this observable itself). In order to avoid this logical pitfall is its preferable that the concept of ionosphere-free linear combinations of observables should be used.

Ionosphere frequencies are varies from experiment to expereiment and even varies during the same experiment. The table below shows typical *cyclic* ionosphere frequencies and ionosphere delays for experiment c1014 (01JUL09XA):

$$\begin{array}{lcllcl} f_{gx} & = & 8.557 \cdot 10^9 \text{ Hz} & f_{gs} & = & 2.293 \cdot 10^9 \text{ Hz} \\ f_{px} & = & 8.215 \cdot 10^9 \text{ Hz} & f_{ps} & = & 2.233 \cdot 10^9 \text{ Hz} \\ \tau_{igx} & = & 18.3 \ \Delta\text{TEC psec} & \tau_{igs} & = & 255.6 \ \Delta\text{TEC psec} \\ \tau_{ipx} & = & 19.9 \ \Delta\text{TEC psec} & \tau_{ips} & = & 269.5 \ \Delta\text{TEC psec} \end{array} \tag{16}$$

3 Implementation in Solve

3.1 Computation of effective ionosphere frequencies

Effective ionosphere frequencies for a specific band are computed each time when a database (but not a superfile) for this band is loaded in Solve. Therefore, S-band effective frequencies are computed usually only once: when the S-band database is loaded, while the X-band effective frequencies are computed each time when a database is loaded or a superfile is created.

Before 2001.07.09 effective ionosphere frequencies were computed under assumption that all channels had equal weights: the same fringe amplitude and the same number of correlated bit instead of using the weights which Fourfit used when it computed group and phase delays. This resulted in errors in ionosphere-free linear combinations used in analysis at the level of 1–10 psec.

This error was corrected on 2001.07.09. Solve versions after that date use the following strategy:

- If the number of processed samples is available then weights which are proportional to a product of the number of processed samples and fringe amplitudes are used;

- If the number of processed samples is not available but the number of used accumulated periods is available, then weights which are proportional to a product of the number of accumulated periods and fringe amplitudes are used;

- If neither this information is available or channel frequencies are unavailable or they have wrong values, then as a last resort unit weights are used.

3.2 Taking into account effects of ionosphere

Modern Solve provides two opportunities to take into account effect of ionosphere on delays:

1. to apply "ionosphere calibration". This quantity is computed either by GAMB when group delay ambiguities are resolved or by IONO. This ionosphere calibration and its formal uncertainty are saved in the database. When "ionosphere calibration" is applied, it is added to theoretical delay, and formal uncertainty of the "calibrated delay" is re-computed. NB: there is a small error in the algorithm for computation of formal uncertainties of "calibrated delay", which gives the value inflated by about 1-3%. This error was not corrected for compatibility with old results.

2. to use ionosphere-free linear combination. Ionosphere-free linear combinations are not saved in the database and are computed on the fly.

These two approaches are almost equivalent. There is a very small difference in results obtained by these two approaches due to the error in computation of formal uncertainties. Another peculiarity of the first approach is that IONO and GAMB before 2001.07.10 locked status flag "suppressed" if the observation at S-band had quality code less than 8 and did not allow to restore such a point without re-running IONO. Since majority of the databases created before 1998 does not keep values of S band group and phase delay observables, we have to use "ionosphere calibration" instead of ionosphere-free linear combinations in batch Solve.

References

[1] Herring, T.A., Precision and accuracy of intercontinental distance determinations using radio interferometry, Ph. D. Thesis, 442 p., 1983