CHAPTER 12 GENERAL RELATIVISTIC MODELS FOR PROPAGATION

VLBI Time Delay

Modeling VLBI time delay follows the first post-Minkowskian approximation of the equations of gravitation field developed by Kopeikin and Schaefer [1].

The formalism of description of light propagation includes three main constituents: (1) equations of the gravitational field, (2) equations of light propagation, and (3) initial and/or boundary conditions. The equations of gravitational field are formulated in harmonic coordinate system with arbitrary origin in space. The equations below are obtained in the linear approximation with respect to the universal gravitational constant G without expansion in the velocities of bodies, so-called first post-Minkowskian approximation. The metric tensor of the gravitational field is found immediately from the energy-momentum tensor of the source of gravitational field described by equations of Einstein [2]. The metric tensor is a function of the retarded time argument t' related to the running time t and to the coordinates $\vec{x}$ of photon, as well as the retarded coordinates $\vec{x}_a(t')$ of the gravitating body by the light-cone equation $t' = t - \frac{1}{c}|\vec{x} - \vec{x}_a(t')|$.

The relativistic model of light propagation assumes that the solar system consists of spherically symmetrical bodies: Sun, Mercury, Venus, Earth, Moon, Mars, Jupiter, Saturn, Uranus and Neptune whose motion is described by the numerical ephemerides DE403. The source is assumed to be a point-like and a radio signal propagates in the medium which may have a refractivity different from 1.

The VLBI delay is modeled as the difference of two intervals: 1) the interval of proper time measured by the clock of the second station between the events: arrival of the radio signal from the source to the reference point of the second station and clock synchronization; 2) the interval of proper time measured by the clock of the first station between the events: arrival of the radio signal from the source to the reference point of the first station and clock synchronization.

Table 12.1. Notation used in the formulas

T_i	—	the BRS time of arrival of a radio signal at the reference point of the i^{th} antenna.
t_i	—	the GRS time of arrival of a radio signal at the reference point of the i^{th} antenna.
t_{ip}	—	the interval of proper time measured by the clock of the i^{th} VLBI station between the event: clock synchronization and arrival of a radio signal to the reference point of the i^{th} antenna.
T'_{ia}	—	retarded BRS time of arrival of a radio signal at the reference point of the i^{th} antenna with respect to the a^{th} body.
T_i^{atm}	—	atmosphere path delay between the source and the reference point of the i^{th} antenna. It includes contribution of ionospheric, neutral hydrostatic and non-hydrostatic constituents.
ΔT_{grav}	—	the differential gravitational time delay.
ΔT_{atm}	—	the differential atmosphere time delay.
$\vec{X}_i(T_i)$	—	the BRS vector from the geocenter to the reference point of the i^{th} antenna at the BRS time T_i.
$\vec{x}_i(t_i)$	—	the GRS vector from the geocenter to the reference point of the i^{th} antenna at the GRS time t_i.
$\vec{b}$	—	$\vec{x}_2(t_1) - \vec{x}_1(t_1)$ the GRS baseline vector at the GRS time t_1 of arrival a radio signal to the

reference point of the first antenna.

$\vec{w}_i$ — the GRS velocity of the reference point of the i^{th} antenna.

$\vec{S}$ — the unit BRS vector from the barycenter to the source.

ρ — the annual parallax of the source.

$\vec{X}_\oplus$ — the BRS vector from the barycenter to the geocenter.

$\vec{V}_\oplus$ — the BRS velocity vector of the geocenter.

$\vec{X}_a$ — the BRS vector from the barycenter to the center of mass of the a^{th} gravitating body.

$\vec{V}_a$ — the BRS velocity vector of the a^{th} gravitating body.

$\vec{R}_{ia}$ — the BRS vector from the a^{th} gravitating body taken at the retarded moment of the BRS time T'_{ia} to the reference point of the i^{th} antenna taken at the moment T_1.

M_a — the mass of the a^{th} gravitating body.

$M_\oplus$ — the mass of the Earth.

$M_\odot$ — the mass of the Sun.

c — the speed of light.

G — the universal gravitational constant.

$a_\oplus$ — the equatorial radius of the Earth.

$\bar{D}_\oplus$ — the mean distance from the geocenter to the barycenter (astronomical unit).

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It is assumed that the BRS time and spatial coordinates are consistent with the BRS metric tensor defined in the Appendix B1.3.2 and the GRS time and spatial coordinates are consistent with the GRS metric tensor defined in the Appendix B1.3.3.

The expression for the VLBI time delay is obtained in three steps: first the difference of barycentric time coordinates is computed.

Omitting the terms with contribution less than 1 psec, the difference of barycentric time coordinates between the events of arrival of radio signal from the point-like source located at the distance >1 parsec from the Earth at the first antenna and at the second antenna, assuming the light is propagating through the medium which causes small additional delay, is expressed according to [1] as:

$$T_2 + T_2^{atm} - T_1 - T_1^{atm} = -\frac{1}{c}\vec{S}\cdot\left(\vec{X}_2(T_2 + T_2^{atm}) - \vec{X}_1(T_1 + T_1^{atm})\right) + \Delta t_{grav} \tag{1}$$

$$\Delta t_{grav} = 2\sum_{a=1}^{N}\frac{GM_a}{c^3}\left(1 + \vec{S}\cdot\vec{V}_a(T'_{1a})\right)\,\ln\frac{|\vec{R}_{1a}| + \vec{S}\cdot\vec{R}_{1a}}{|\vec{R}_{2a}| + \vec{S}\cdot\vec{R}_{2a}} \tag{2}$$

where $\vec{R}_{1a} = \vec{X}_1(T_1) - \vec{X}_a(T'_{1a})$, $\qquad \vec{R}_{2a} = \vec{X}_2(T_1) - \vec{X}_a(T'_{1a})$.

$\vec{X}_a$ is a position of the a^{th} gravitating body taken at a retarded time T'_a which is a solution of the gravitational null-cone equation:

$$T'_{1a} = T_1 - \frac{1}{c}\big|X(T_1) - X_a(T'_{1a})\big| \tag{3}$$

Neglecting terms of $O\big(\frac{\dot{X}_a^2}{c^2}\big)$ and $O\big(\frac{\vec{X}_a \cdot \ddot{\vec{X}}_a}{c^2}\big)$, the retarded moment of time T'_{1a} can be computed precisely enough to provide the accuracy of computation of the gravitation delay better than 1 psec with using the following expression:

$$T'_{1a} = T_1 - \frac{1}{c}\,\frac{|X(T_1) - X_a(T_1)|}{1 - \frac{\vec{V}_a(T_1)}{c}\,\frac{\vec{X}(T_1)}{|\vec{X}(T_1)|}} \tag{4}$$

At the second step the time difference and the vectors of station coordinates are transformed from the barycentric reference frame to the geocentric reference frame by using the expression listed in the Appendix B1.3.4. At the third step the time difference of geocentric time coordinates is transformed to the differences of the intervals of proper time by integrating the expression for time transformation in the Appendix B1.3. The final expression for the VLBI time delay defined above for a source located at the distance greater than 1 parsec is as follows:

$$\begin{aligned} t_{1p} - t_{2p} = & \frac{1}{1 + \frac{\vec{S}\cdot(\vec{V}_\oplus + \vec{w}_2)}{c}} \Bigg(-\frac{\vec{S}\cdot\vec{b}}{c}\left[1 - 2\frac{GM_\odot}{|\vec{X}_\oplus|c^2} - \frac{GM_\oplus}{|a_\oplus|c^2} - \frac{|\vec{V}_\oplus|^2}{2c^2} - \frac{\vec{V}_\oplus\cdot\vec{w}_2}{c^2}\right] - \\ & \frac{\vec{V}_\oplus\cdot\vec{b}}{c^2}\big(1 + \frac{\vec{S}\cdot\vec{V}_\oplus}{2c}\big) + \frac{1}{c}\,\rho\,\vec{b}\,\frac{\vec{R}_\oplus}{\bar{D}_\oplus} + \Delta T_{grav} + \Delta T_{atm} \Bigg) \end{aligned} \tag{5}$$

The differences between the current expression for time delay and the expression in the IERS Conventions 1996 are:

— the IERS Conventions 1996 has the coefficient -2 in front of the term $GM_\oplus/|a_\oplus|c^2$, while the current formula (5) has the coefficient -1. This error, which can be as large as 28 psec, results in scaling the estimates of positions of the VLBI stations obtained from the analysis with using that formula by the factor of $1 + 7\cdot 10^{-10}$.

— IERS Convention 1996 proposed to use positions of the bodies for computation of the gravitation delay at the moment of time when the distance between the center of mass of the body and the photon is minimum while the current expression requires to use the retarded moment of time. This difference can result in the diffrence in VLBI delay greater than 1 psec at the observations closer than 30 arcmin from the Jupiter.

— IERS Conventions 1996 did not multiply atmosphere delay by the factor of $\frac{1}{1+\frac{\vec{S}\cdot(\vec{V}_\oplus+\vec{w}_2)}{c}}$. This effect can exceed 10 psec.

— The last term in formula 10 in the IERS Conventions 1996 is unnecessary since it cannot exceed 0.7 psec.

References

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